A Coupled Oscillators-based Control Architecture for Locomotory Gaits

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Mechanical SCIENCE AND ENGINEERING

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Outline

1. Problem overview
2. Literature survey
3. Proposed approach
4. Example with simulation result
5. Summary

Locomotion

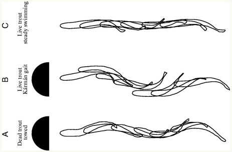
Animal locomotion

Tadpole, San Diego zoo

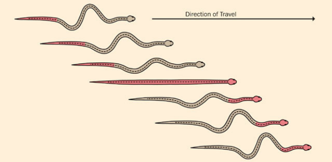
Snake, BBC News

Usain Bolt, theconsultant.eu

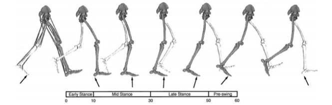
Locomotion gait



Swimming gait, Journal of Experimental Biology



Snake locomotion, Biokids, Univ of Michigan



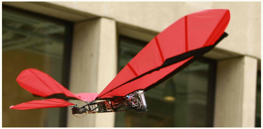
Walking Gait, Science Direct

- P. Holmes, R. J. Full, D. Koditschek, and J. Guckenheimer. The dynamics of legged locomotion, 2006

Bio-Inspired Robots

RHex robot, Boston Dynamics

Snakelike Robot, Biorobotics CMU



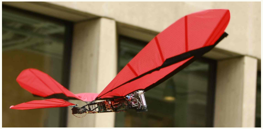
http://groups.csail.mit.edu/locomotion/

Wind up toy robot

- D. Xinyan, L. Schenato, and S. S. Sastry, 2006
- Z. G. Zhang, N. Yamashita, M. Gondo, A. Yamamoto, and T. Higuchi, 2008
- R. L. Hatton and H. Choset, 2010

Periodic actuation of internal degree of freedom $\rightarrow$ global displacement

RHex robot, Boston Dynamics



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Shape and Group Variable

Shape variable

$x \in M$

$x = (x_1, x_2) \in T^2$

Internal dynamics

$\ddot{x} = F(x, \dot{x}, \tau)$

$I(x)\ddot{x} = C(x, \dot{x})\dot{x} - kx + \tau$

Group variable

$g \in G$

$g = (\vec{r}, \psi) \in SE(2)$

Group dynamics

$g^{-1}\dot{g} = A(x)\dot{x}$

$\dot{\psi} = A_1(x_1, x_2)\dot{x}_1 + A_2(x_1, x_2)\dot{x}_2$

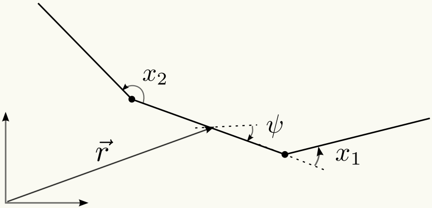

Figure : 3-link system

- S. D. Kelly, and R. M. Murray, Geometric Phase and Robotic Locomotion, 1994

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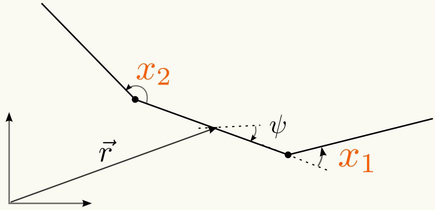

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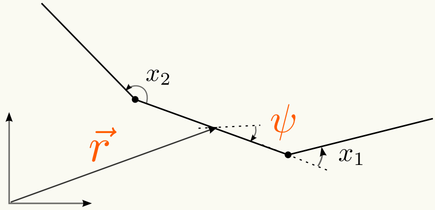

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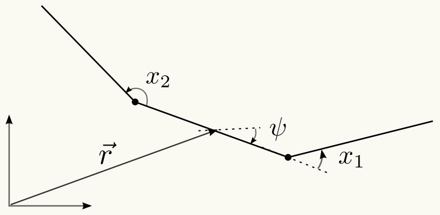

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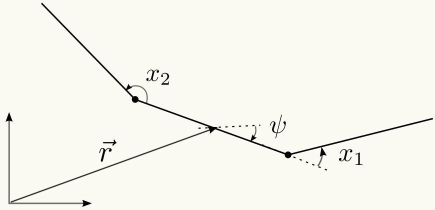

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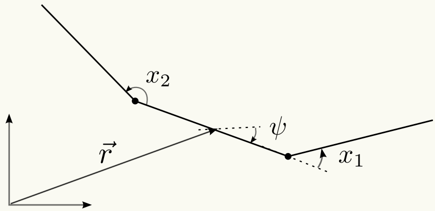
The dynamics does not dependen on the group variable



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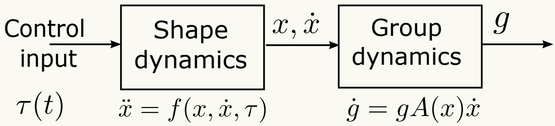
General Approach



Approach

1. **Gait design:** Choose a periodic orbit in the shape space to induce the desired change in group variable.
2. **Gait generation:** Implement a law for the control input, that leads to the desired periodic orbit in the shape space.

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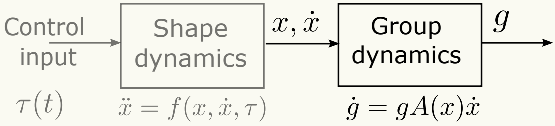

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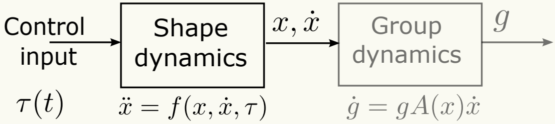

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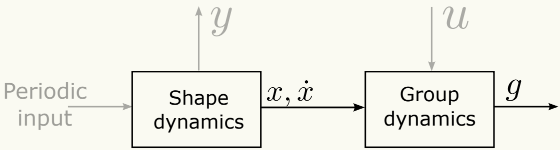
Proposed Approach

Approach

1. Periodic input, to have shape variable oscillate in periodic manner.
2. Noisy sensory measurements of the shape variables.
3. Control actuation via manipulation of parameters of the system.
4. Find optimal control law, to achieve maneuver about nominal gait, based on noisy sensory measurements



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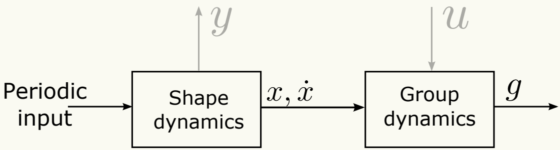

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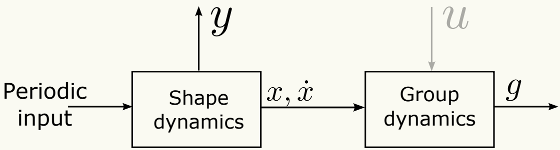

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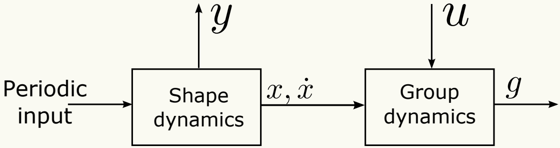

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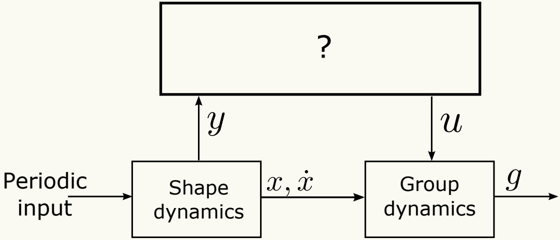
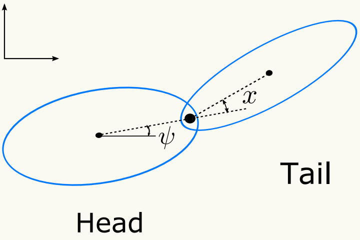


Example: 2-body System

Dynamics:

$$M(x)\ddot{x} = C(x)\dot{x}^2 + B(x)(\tau(t) - \kappa x - b\dot{x})$$
$$\dot{\psi} = \tilde{A}(x)\dot{x}$$



Shape variable: x

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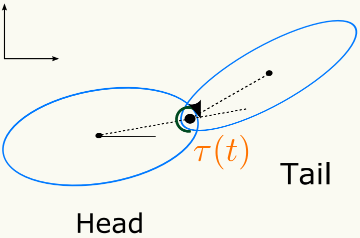

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$$\tau(t) = \tau_0 \sin(\omega_0 t)$$

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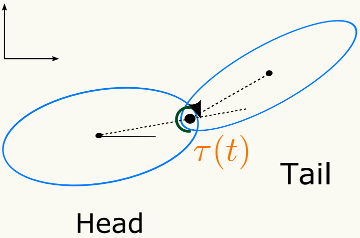

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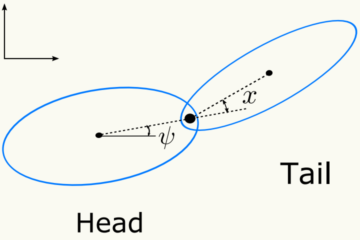
Measurement:

$$\mathrm{d}Z_t = \tilde{h}(x,\dot{x})\,\mathrm{d}t + \mathrm{d}W_t$$

$W(t)$: Wiener process



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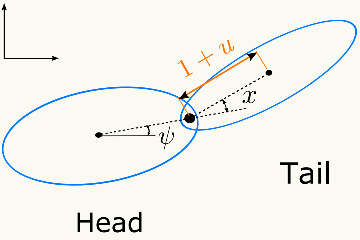
Control actuation:

$$d(t) = \bar{d}(1+u) \quad \rightarrow \quad \dot{\psi} = A(x,u)\dot{x}$$



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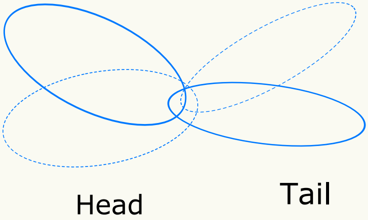
$$\min_{u_{[0,T]}} \mathsf{E}\left[(\psi(T) - \psi(0)) + \frac{1}{2\varepsilon}\int_0^T u(t)^2 \mathrm{d}t\right]$$

- Geometric phase
- Control cost
- Small control penalty parameter



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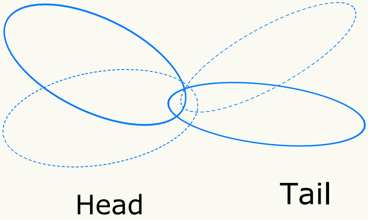

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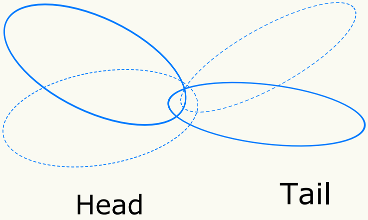

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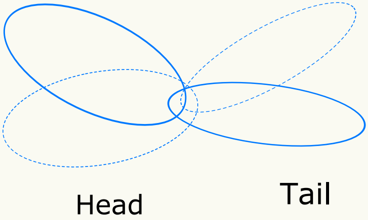

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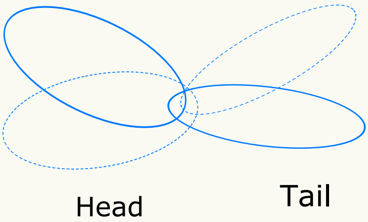

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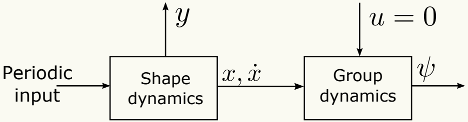
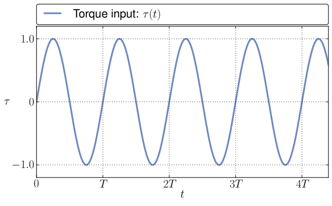
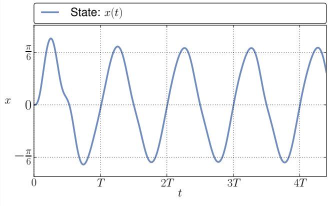
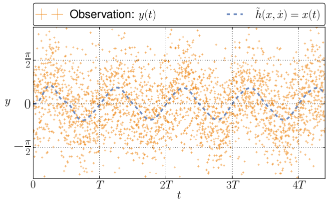
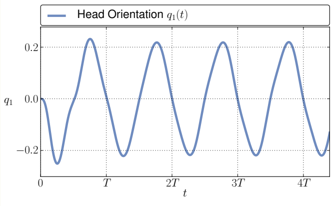
Numerical Result, Open Loop











Solution: Phase reduction

Dynamics:

$$M(x)\ddot{x} = C(x)\dot{x}^2 + B(x)(\tau(t) - \kappa x - b\dot{x})$$
$$\dot{\psi} = \tilde{A}(x,u)\dot{x}$$

Periodic control input:

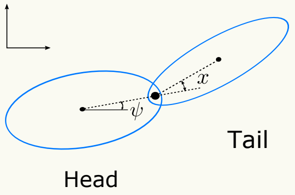
$$\tau(t) = \tau_0 \sin(\omega_0 t)$$

Measurement:

$$\mathrm{d}Z_t = \tilde{h}(x,\dot{x})\,\mathrm{d}t + \mathrm{d}W_t$$
$$W(t): \quad \text{Wiener process}$$

Limit cycle solution:

$$X_{LC}(\theta(t)) = (x(t), \dot{x}(t))$$
$$\theta(t) = (\omega_0 t + \theta_0) \text{ mod } 2\pi$$



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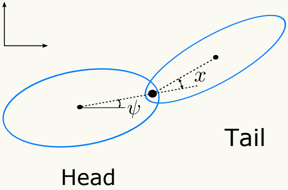

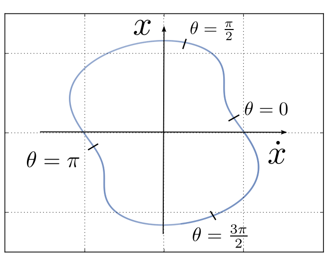

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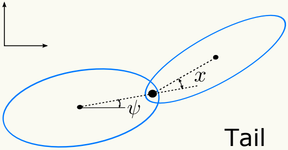

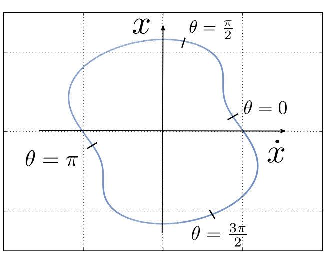

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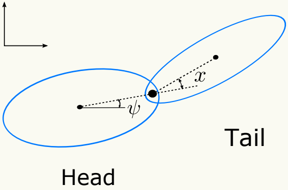

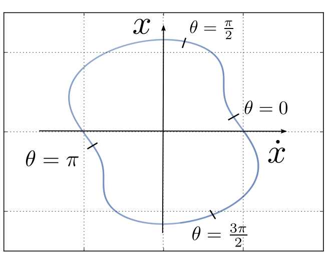

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Periodic control input:

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Measurement:

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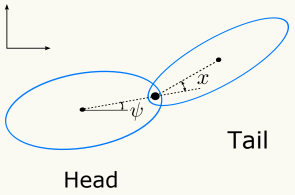

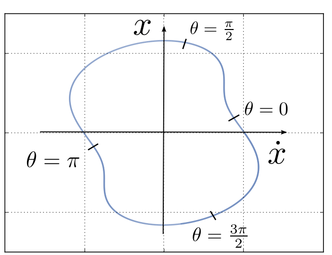

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Measurement:

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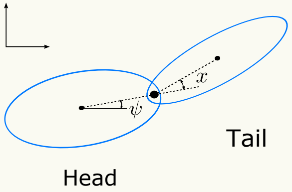
$W(t)$: Wiener process

Objective: Turning the head,

$$\min_{u_{[0,T]}} \mathsf{E}\left[(\psi(T) - \psi(0)) + \frac{1}{2\varepsilon}\int_0^T u(t)^2 \mathrm{d}t\right]$$



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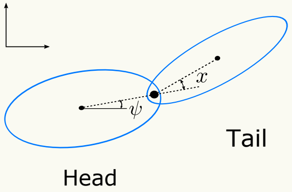

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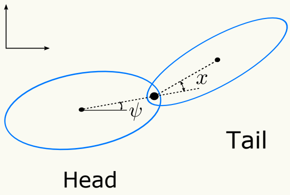
$W(t)$: Wiener process



Objective: Turning the head,

$$\min_{u_{[0,T]}} \mathsf{E}\left[(\psi(T) - \psi(0)) + \frac{1}{2\varepsilon}\int_0^T u(t)^2 \mathrm{d}t\right] =$$

$$\min_{u_{[0,T]}} \mathsf{E}\left[\int_0^T \left(A(\theta(t), u(t)) + \frac{1}{2\varepsilon} u(t)^2\right) \mathrm{d}t\right]$$

Solution: Optimal Control

Dynamics:

$$M(x)\ddot{x} = C(x)\dot{x}^2 + B(x)(\tau(t) - \kappa x - b\dot{x})$$

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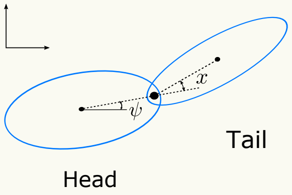

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$$u^*(t) = -\varepsilon \mathsf{E}\Big[\frac{\partial A}{\partial u}(\theta(t), u^*(t)) \mid \mathscr{Z}_t\Big]$$

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Dynamics:

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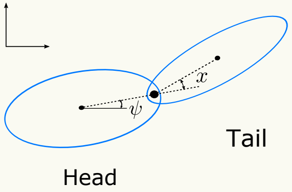

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$$u^*(t) = -\varepsilon \mathsf{E}\left[\frac{\partial A}{\partial u}(\theta(t), u^*(t)) \mid \mathscr{Z}_t\right]$$

Construct filter to evaluate the average

Sloution: Coupled Oscillator Feedback Particle Filter

$$\textbf{Signal:} \quad \mathrm{d}\theta_t = \omega_0 \,\mathrm{d}t + \mathrm{d}B_t, \quad \bmod 2\pi$$

$$\textbf{Observations:} \quad \mathrm{d}Z_t = h(\theta_t)\,\mathrm{d}t + \mathrm{d}W_t$$

- Yang, Mehta, Meyn. Feedback Particle Filter. IEEE Trans. Automatic Control (Oct 2013).
- Laugesen, Mehta, Meyn, Raginsky. Poisson's Equation in Nonlinear Filtering. SIAM J. Opt. Control (2014).

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Problem: Estimate the phase θ_t from noisy observations.

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$$\textbf{Observations:} \quad \mathrm{d}Z_t = h(\theta_t)\,\mathrm{d}t + \mathrm{d}W_t$$

Problem: Estimate the phase θ_t from noisy observations.

$$\textbf{FPF:} \quad \mathrm{d}\theta_t^i = \ \omega^i\,\mathrm{d}t \ + \ \mathrm{d}B^i(t) \ + \ \mathsf{K}(\theta^i,t)\circ\Big(\mathrm{d}Z_t - \tfrac{1}{2}(h(\theta_t^i)+\hat{h}_t)\,\mathrm{d}t\Big)$$

$$i = 1,\ldots N, \quad \text{mod } 2\pi$$

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- i^{th} oscillator's frequency

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- i^{th} oscillator's noise

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- i^{th} oscillator's control

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Observations: $\mathrm{d}Z_t = h(\theta_t)\,\mathrm{d}t + \mathrm{d}W_t$

Problem: Estimate the phase θ_t from noisy observations.

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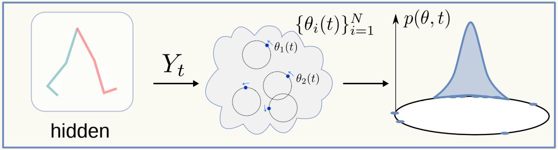

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Dynamics:

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$W(t)$: Wiener process

Objective:

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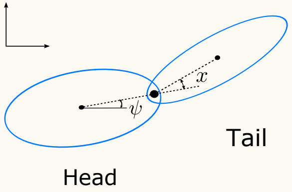

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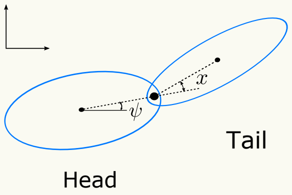
$$-\varepsilon \frac{1}{N}\sum_{i=1}^N \frac{\partial A}{\partial u}(\theta^i(t), u^*(t))$$



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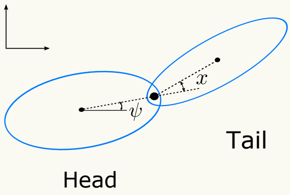
$$-\varepsilon \frac{1}{N}\sum_{i=1}^{N} \frac{\partial A}{\partial u}(\theta^i(t), u^*(t)) = -\varepsilon \frac{1}{N}\sum_{i=1}^{N} \frac{\partial A}{\partial u}(\theta^i(t), 0) + O(\varepsilon^2)$$



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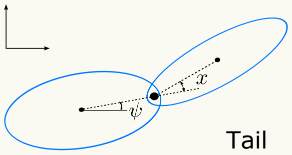

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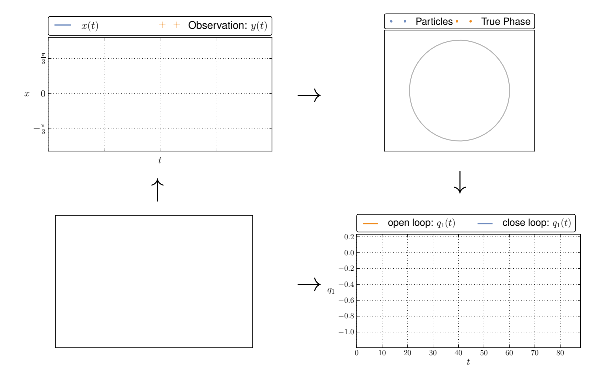
$$-\varepsilon \frac{1}{N}\sum_{i=1}^{N} \frac{\partial A}{\partial u}(\theta^i(t), u^*(t)) = -\varepsilon \frac{1}{N}\sum_{i=1}^{N} \frac{\partial A}{\partial u}(\theta^i(t), 0) + O(\varepsilon^2) \approx -\varepsilon \frac{1}{N}\sum_{i=1}^{N} \phi(\theta^i)$$

2-body system, Simulation Result



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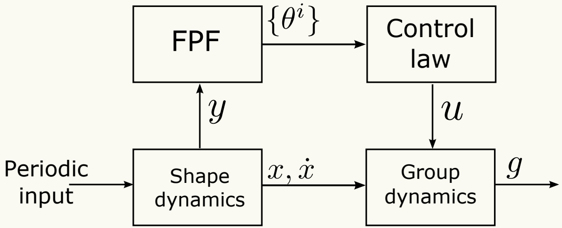
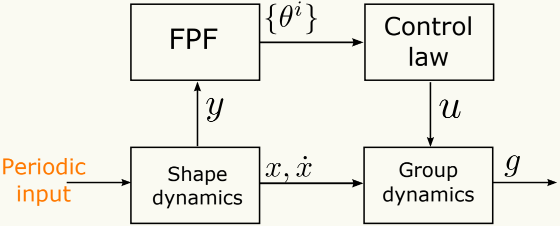
In Summary



- Periodic input
- Noisy sensory measurements of the shape
- Coupled oscillator Feedback Particle Filter to estimate the shape
- Optimal control law based on oscillators
- Maneuver around a nominal gait

In Summary



- Periodic input
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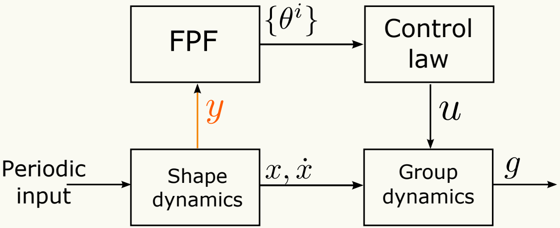

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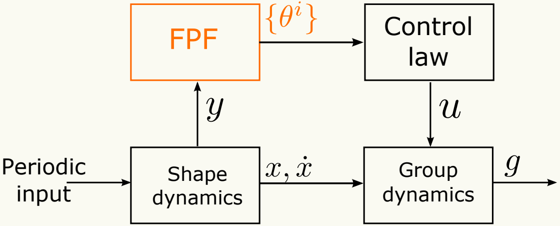

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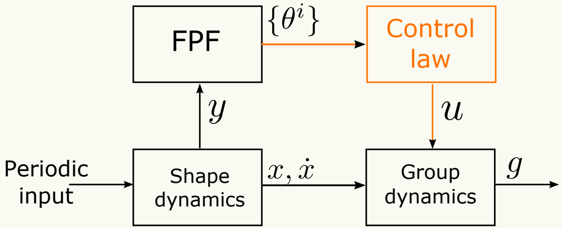

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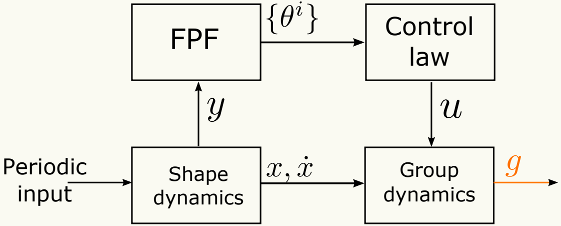

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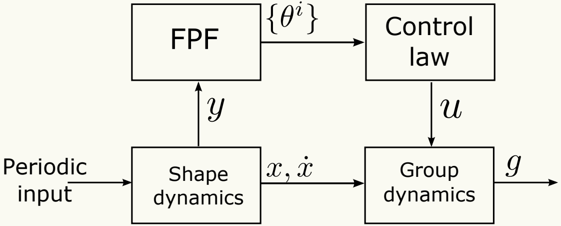

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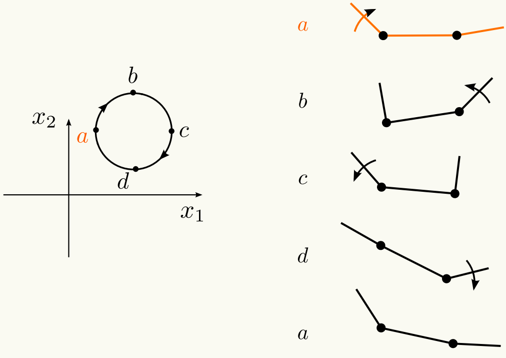
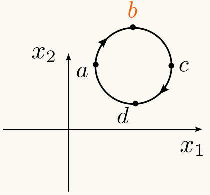
Thank You

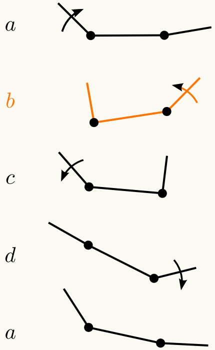
Questions?



- R. W .Brockett. Patterngenerationandthecontrolofnonlinearsystems. 2004
- P. S. Krishnaprasad. Geometric phases, and optimal reconfiguration for multibody systems, 1994

Rectification, Geometric Phase





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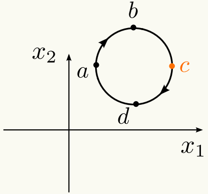

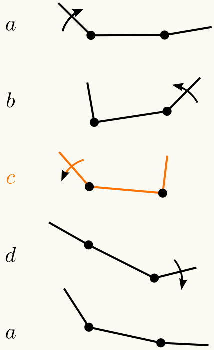

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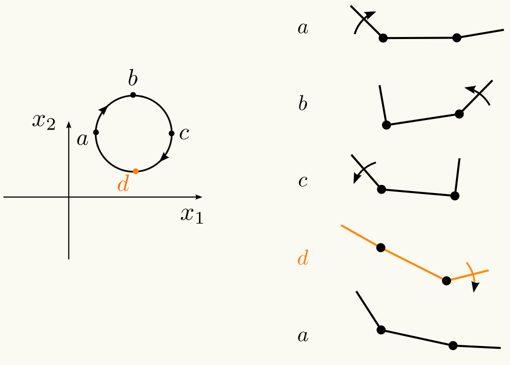

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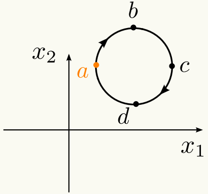

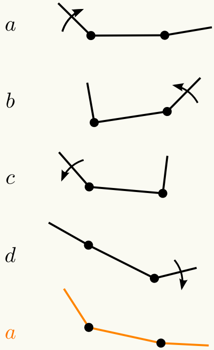

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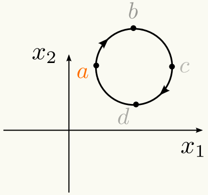
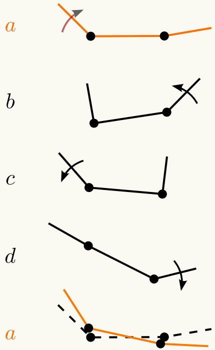
Geometric Phase: Net change in group variable over one cycle





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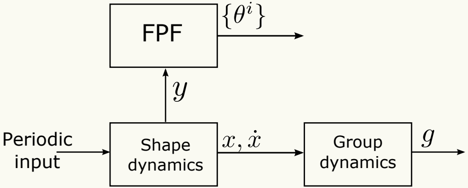
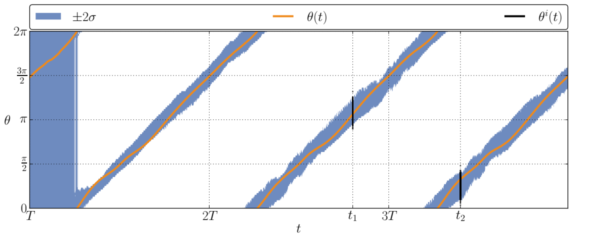
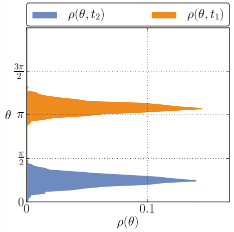
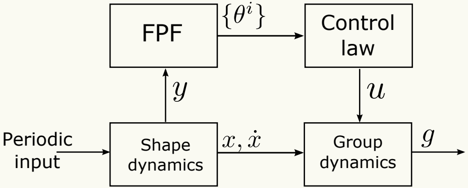
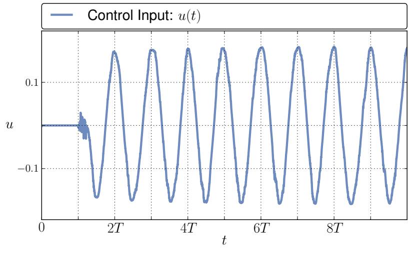
Numerical Result, Estimation

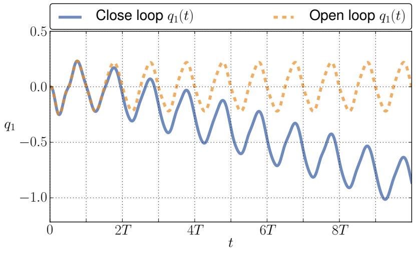







FPF
$\{\theta^i\}$
Control law
y
u
Periodic input
Shape dynamics
$x, \dot{x}$
Group dynamics
g


Control Input: $u(t)$
u
0.1
−0.1
0
2T
4T
6T
8T
t


Close loop $q_1(t)$
Open loop $q_1(t)$
q_1
0.5
0.0
−0.5
−1.0
0
2T
4T
6T
8T
t