Feedback Particle Filter: Design, Approximation, and Error Analysis

Presented at the University of California Irvine

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Ph.D. advisor: Prashant G. Mehta

Coordinated Science Laboratory
University of Illinois at Urbana-Champaign

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ILLINOIS

Research Overview

Nonlinear filtering (this talk)

- A. Taghvaei, J de Wiljes, P. G. Mehta, and S. Reich. Kalman filter and its modern extensions for the continuous-time nonlinear filtering problem. ASME, 2017

Sampling

- A. Taghvaei, P. G. Mehta. Accelerated gradient flow for probability distributions, ICML, 2019

Deep learning

- A. Taghvaei, J. Kim, P. G. Mehta. How regularization affects the critical points in linear neural networks, NIPS, 2017

Common theme

- Control problem on probability distribution
- Interacting particle system as algorithm

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Outline

- Part I: Background
  - Filtering problem
  - Kalman filter (1960's)
  - Ensemble Kalman filter & particle filter (1990s)
- Part II: Feedback Particle Filter (2013-)
  - Design
  - Approximation
  - Error analysis

Message of part I:

Kalman filter $\longrightarrow$ Ensemble Kalman filter $\longrightarrow$ Feedback Particle filter

and importance controlled interacting particle systems

Message of part II: How to analyze these systems

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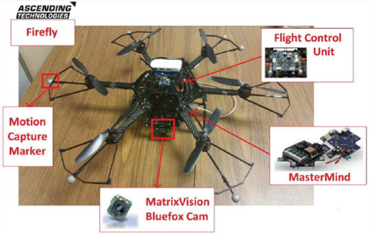

Observation

Hidden state: Position and orientation of quadrotor

Observation: Camera, GPS, and motion sensor

Problem: Estimate the state based on observation

Filtering approach: Compute the conditional probability distribution

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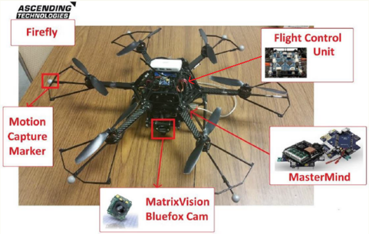

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Dynamical system:

State process: $\mathrm{d}X_t = \text{(dynamical model)}, \qquad X_0 \sim p_0(\cdot)$

Observation process: $\mathrm{d}Z_t = h(X_t)\,\mathrm{d}t + \mathrm{d}W_t$

Filtering objective: Compute the posterior distribution $\mathsf{P}(X_t | Z_{[0,t]})$

Solution:

- In principle, Bayes rule. In practice, impossible to implement it
- Linear Gaussian setting: Kalman filter
- General setting: Approximate solutions

J. Xiong, An introduction to stochastic filtering theory, 2008

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- linear observation model: $h(x) = Hx$

Kalman-Bucy filter: $P(X_t|\mathcal{Z}_t)$ is Gaussian $\mathcal{N}(m_t, \Sigma_t)$

Update for mean: $$\mathrm{d}m_t = \underbrace{Am_t\,\mathrm{d}t}_{\text{dynamics}} + \underbrace{\mathsf{K}_t(\mathrm{d}Z_t - Hm\,\mathrm{d}t)}_{\text{correction}}$$

Update for covariance: $$\frac{\mathrm{d}\Sigma_t}{\mathrm{d}t} = \text{(Ricatti equation)}$$

Kalman gain: $$\mathsf{K}_t := \Sigma_t H^\top$$

Properties

- Close relation to optimal control theory
- Strong results about the stability of the filter

Question: What is the generalization to the nonlinear and non-Gaussian setting?

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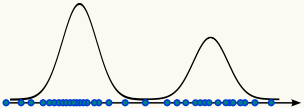
- Filtering problem has no finite-dim. solution in general $\rightarrow$ approximations
- Monte-Carlo method: Approximate with empirical distribution of N particles

$$P(X_t) \approx \frac{1}{N}\sum_{i=1}^{N} \delta_{X_t^i}$$



Example:

State process: $\mathrm{d}X_t = a(X_t)\,\mathrm{d}t + \sigma(X_t)\,\mathrm{d}B_t$

Objective: compute $\mathsf{P}(X_t)$

Monte-Carlo method: Simulate N independent samples

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Question: Can we generalize this idea to the filtering problem?

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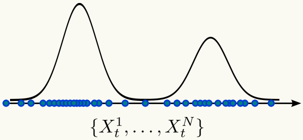

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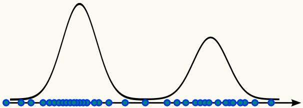

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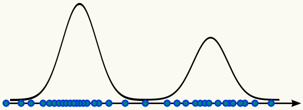

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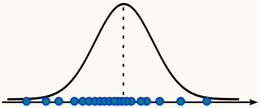

Properties

- In the limit ($N = \infty$), the mean and variance evolve according to the Kalman filter
- Computational complexity is $O(Nd)$, efficient when $d \gg N$
- It is not exact in a general setting

Question: What is the generalization of EnKF in a general setting?

G. Evensen. Sequential data assimilation with a nonlinear quasi-geostrophic model . . . 1994.
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Ensemble Kalman filter

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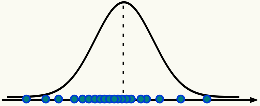

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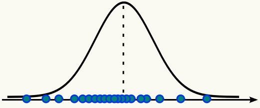

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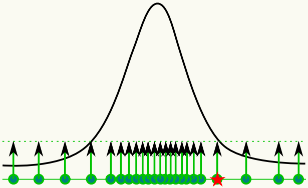
Idea: Approximate the posterior $P(X_t|\mathcal{Z}_t)$ using weighted dist. of particles $\{X_t^i, W_t^i\}_{i=1}^N$

$$P(X_t|\mathcal{Z}_t) \approx \sum_{i=1}^{N} W_t^i \delta_{X_t^i}$$

Update the wights based on the likelihood model (importance sampling)



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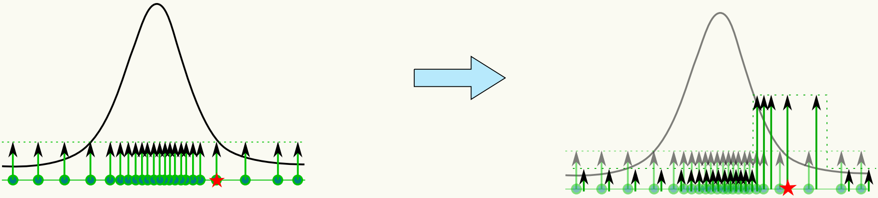

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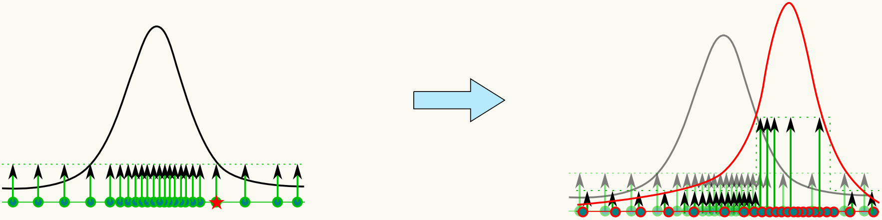

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Particle filter
Sequential Monte-Carlo method

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Problems:

1. Particle impoverishment for high-dimensional problems – $N \propto \exp(d)$
2. No control structure
3. No relation to Ensemble Kalman filter

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Controlled interacting particle system

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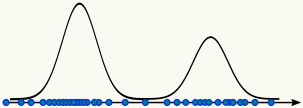

Update formula:

$$\mathrm{d}X_t^i = (\text{dynamic model}) + \underbrace{\mathsf{K}_t(X_t^i) \circ \left(\mathrm{d}Z_t - \frac{h(X_t^i) + N^{-1}\sum_{j=1}^{N} h(X_t^j)}{2}\mathrm{d}t\right)}_{\text{correction}}, \quad X_0^i \overset{\text{i.i.d}}{\sim} p_0$$

Gain function: $\mathsf{K}_t(x) = \nabla\phi_t(x)$ where ϕ is the solution to a PDE

Properties

- Exact in the limit $N = \infty$
- Feedback control structure
- Simplify to EnKF in linear Gaussian setting

T. Yang, R. S. Laugesen, P. G. Mehta, and S. P. Meyn. Multivariable feedback particle filter, *Automatica*, 2016

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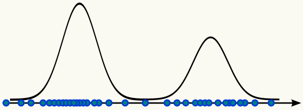

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Recap:

KF: $\mathrm{d}m_t = \text{(dynamical model)} + \mathsf{K}_t(\,\mathrm{d}Z_t - Hm_t\,\mathrm{d}t)$

EnKF: $\mathrm{d}X_t^i = \text{(dynamical model)} + \mathsf{K}_t^{(N)}(\,\mathrm{d}Z_t - \frac{HX_t^i + N^{-1}\sum_{j=1}^N HX_t^j}{2}\,\mathrm{d}t)$

FPF: $\mathrm{d}X_t^i = \text{(dynamical model)} + \mathsf{K}_t^{(N)}(X_t^i) \circ (\,\mathrm{d}Z_t - \frac{h(X_t^i) + N^{-1}\sum_{j=1}^N h(X_t^j)}{2}\,\mathrm{d}t)$

Analysis of FPF:

Design	Approximation	Error analysis
Mean-field limit $\bar{X}_t, \bar{U}_t$	$\longrightarrow$	Finite-N system $\{X_t^i, U_t^i\}_{i=1}^N$

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Design	**Approximation**	**Error analysis**
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  - Monte-Carlo approximation: Ensemble Kalman filter & particle filters (1990s)
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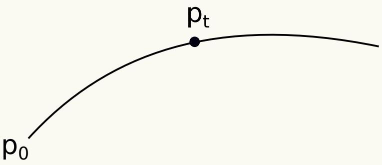
Design: problem overview



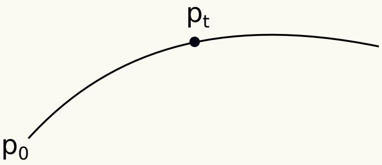
The trajectory of the posterior distribution $p_t := \mathsf{P}(X_t|Z_{[0,t]})$ in the probability space

Design problem: Construct a random process $\bar{X}_t$ that follows the posterior, i.e

$$\bar{X}_t \sim p_t, \quad \forall t \geq 0$$

Non-uniqueness: There are infinitely many solutions

Design: problem overview



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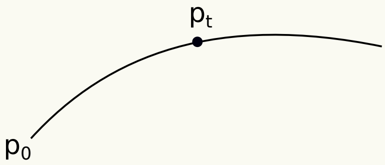

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Example:

State process: $\mathrm{d}X_t = \mathrm{d}B_t, \quad X_0 \sim N(0,1)$

Objective: compute $\mathsf{P}(X_t)$

Two solutions:

(I) $\mathrm{d}X_t^i = \mathrm{d}B_t^i$

(II) $\frac{\mathrm{d}}{\mathrm{d}t} X_t^i = \frac{X_t^i}{\frac{2}{N}\sum_{j=1}^N (X_t^i)^2}$

They both produce the same distribution $N(0, 1+t)$.

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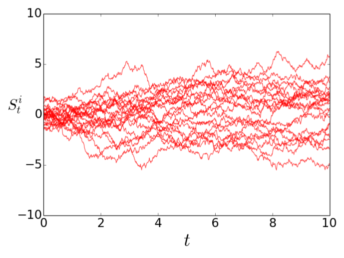

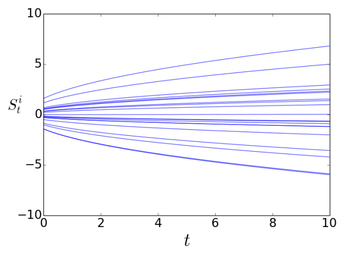

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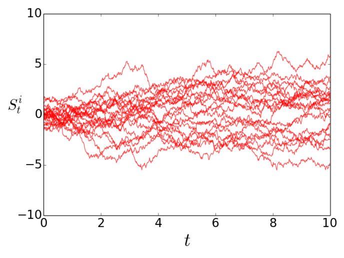

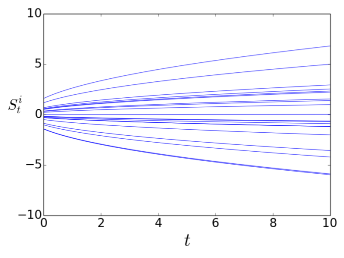

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Optimal transportation approach

- Reason for non-uniqueness: Only the marginal distributions, at each time instant, are specified
- Optimal transport maps provide a way to uniquely couple two distributions.

Proposed solution: Infinitesimal optimal transport maps

$$\bar{X}_{t+\Delta t} = T_t(\bar{X}_t),$$

- T_t is the optimal transport map between p_t and $p_{t+\Delta t}$
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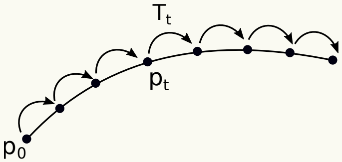

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Proposition

In linear Gaussian setting, the optimal transportation procedure result in the following process:

$$\mathrm{d}\bar{X}_t = \text{(terms effecting the mean)} + G_t(\bar{X}_t - \bar{m}_t)\,\mathrm{d}t$$

where G_t is the unique symmetric solution to the Lyapunov equation

$$G_t\bar{\Sigma}_t + \bar{\Sigma}_t G_t = \text{Ricc}(\bar{\Sigma}_t)$$

Comparison: A non-optimal (and stochastic) solution is

$$\mathrm{d}\bar{X}_t = \text{(terms effecting the mean)} + (A - \frac{1}{2}\bar{\Sigma}_t H^\top H)(\bar{X}_t - \bar{m}_t)\,\mathrm{d}t + \sigma_B\,\mathrm{d}\bar{B}_t$$

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A. Taghvaei, P. G. Mehta, An optimal transport formulation for the linear feedback particle filter, (ACC) 2016

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FPF update formula:

$$\mathrm{d}X_t^i = \text{(dynamic model)} + \mathsf{K}_t(X_t^i) \circ (\,\mathrm{d}Z_t - \frac{1}{2}(h(X_t^i) + \hat{h}_t)\,\mathrm{d}t)$$

Gain function $\quad \mathsf{K}_t(x) = \nabla\phi_t(x) \quad$ where ϕ solves the Poisson eq.

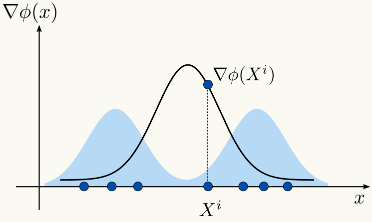
Poisson equation:

$$-\frac{1}{p_t(x)}\nabla\cdot(p_t(x)\nabla\phi_t(x)) = h(x) - \hat{h}_t$$

Computational problem:

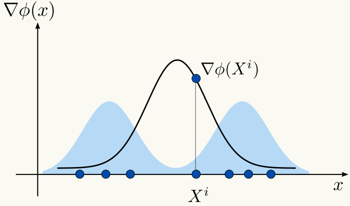
Given: $\{X_t^1,\ldots,X_t^N\} \overset{\text{i.i.d}}{\sim} p_t$

Approximate: $\{\mathsf{K}_t(X_t^1),\ldots,\mathsf{K}_t(X_t^N)\}$



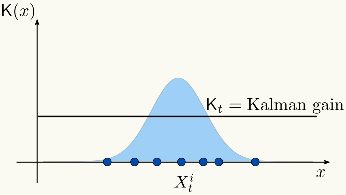
General setting

$\mathsf{K}(x) = ?$



Linear Gaussian setting

$\mathsf{K}(x) = \mathsf{K} \quad$ (Kalman gain)



$$\text{FPF} \xrightarrow{\text{const. gain}} \text{EnKf}$$

A. Taghvaei, J de Wiljes, P. G. Mehta, and S. Reich. Kalman filter and its modern extensions for the continuous-time nonlinear filtering problem. ASME, 2017

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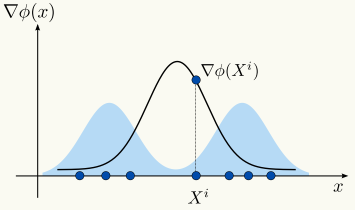

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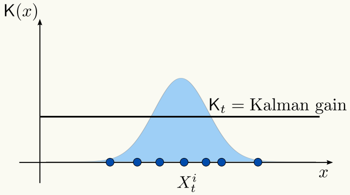

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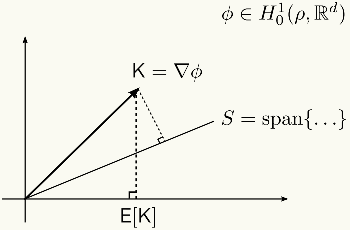

Choice of basis function is difficult

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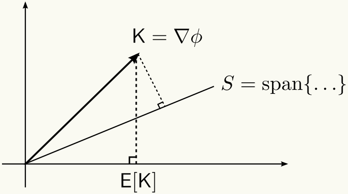
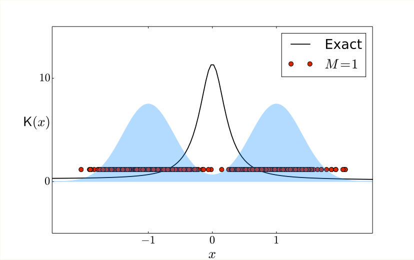
$\phi \in H_0^1(\rho, \mathbb{R}^d)$





$$S = \text{span}\{1, x, \ldots, x^M\}$$

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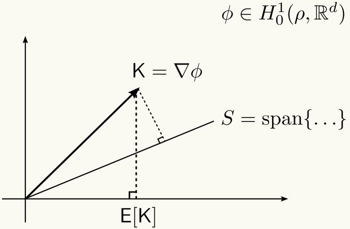

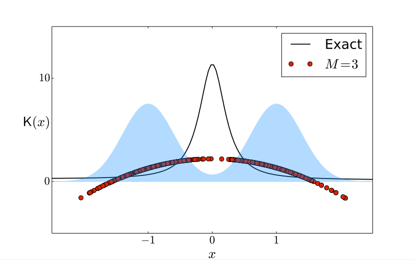

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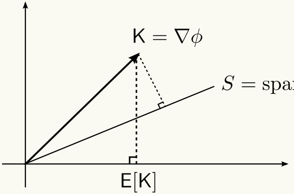

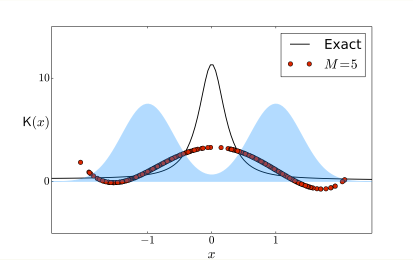

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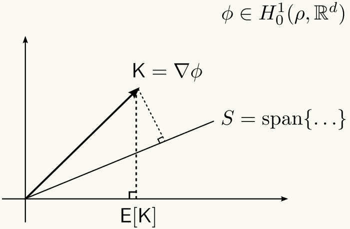

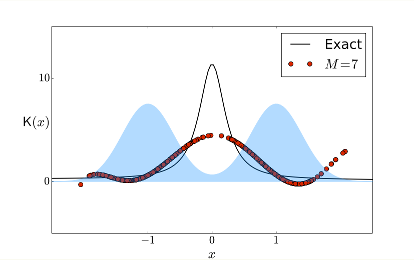

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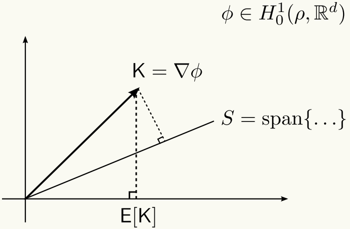

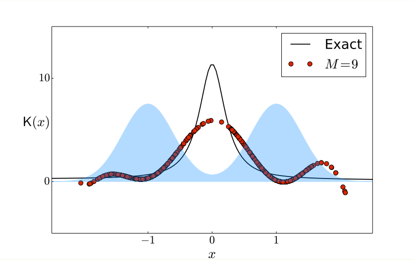

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Galerkin approximation

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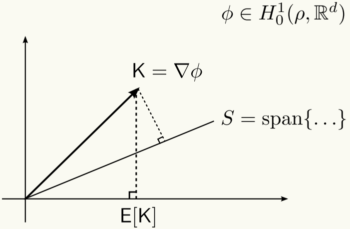

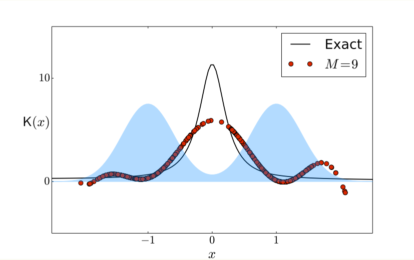

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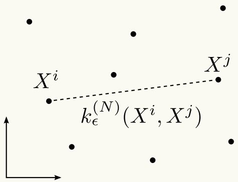
- **Stochastic formulation:**

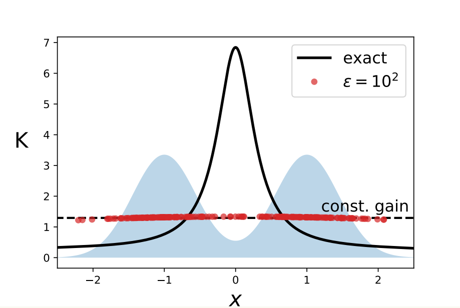
$$\phi = P_\epsilon \phi + \int_0^\epsilon P_s(h - \hat{h})\,\mathrm{d}s$$

where $\{P_t\}$ is the semigroup for $\Delta_\rho := \frac{1}{\rho}\nabla \cdot (\rho \nabla)$

- Approximate P with a Markov matrix using particles (Coifman & Lafon, 2006)



- The resulting approximation takes the form $\mathsf{K}(X^i) \approx \sum_{j=1}^{N} s_{ij} X^j$



Error estimates: $\text{r.m.s.e} = \underbrace{O(\epsilon)}_{\text{bias}} + \underbrace{O(\frac{1}{\epsilon^{1+d/2} N^{1/2}})}_{\text{variance}}$

Question: How does the error effect the distribution?

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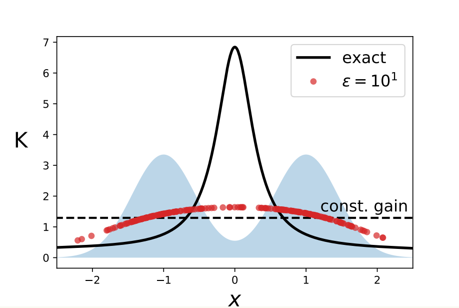

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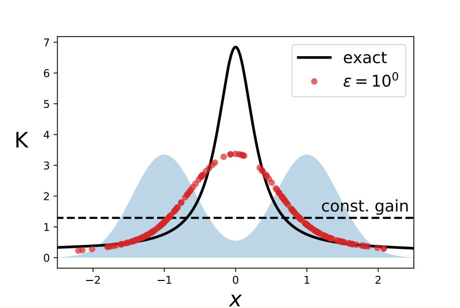

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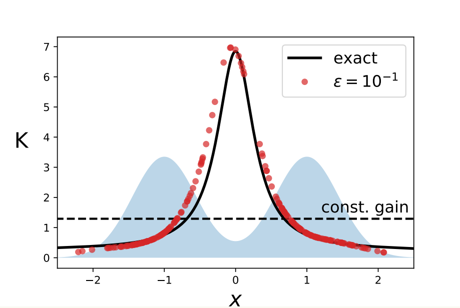

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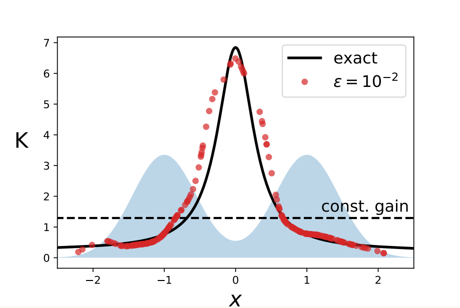

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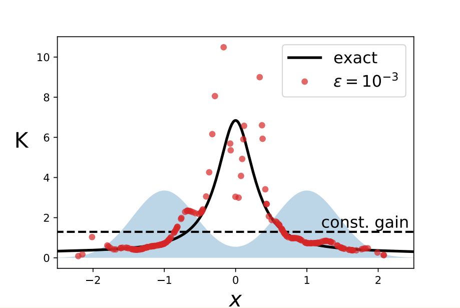

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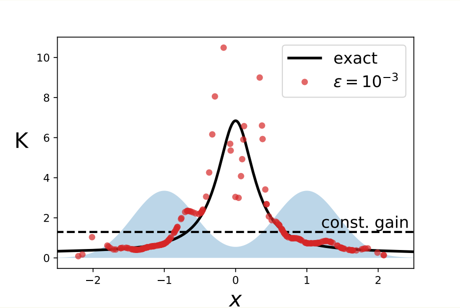

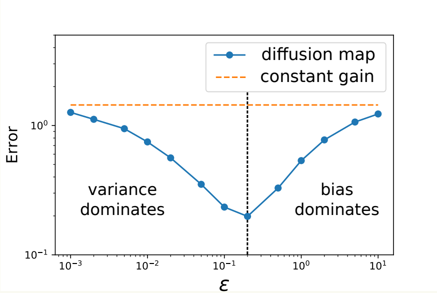

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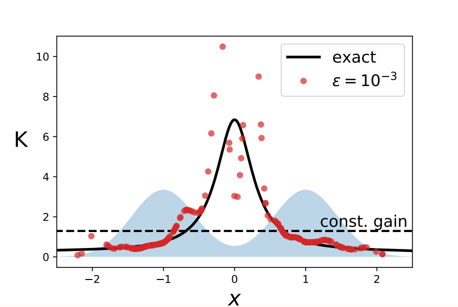

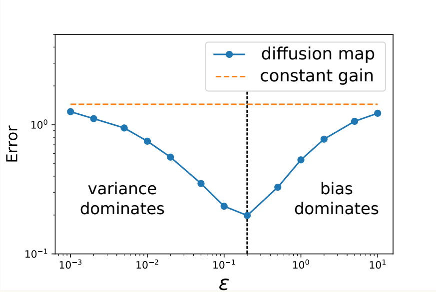

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- Simulating Kalman filter is computationally expensive for high-dimensional problems

$$\begin{aligned}\text{if state dimension is } d \quad &\Rightarrow \quad \text{covariance matrix is } d \times d \\ &\Rightarrow \quad \text{computational complexity is } O(d^2) \\ &\Rightarrow \quad \text{Not scalable for high-dim problems} \\ &\qquad\ \ \text{(e.g weather prediction)}\end{aligned}$$

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$$\mathrm{d}X_t^i = \text{(linear dynamics)} + \mathsf{K}_t^{(N)}(\,\mathrm{d}Z_t - \frac{1}{2}H(X_+^i m_t^{(N)})\,\mathrm{d}t), \quad X_0^i \overset{\text{i.i.d}}{\sim} p_0$$

$$\mathsf{K}_t^{(N)} = \Sigma_t^{(N)} H^\top$$

with empirical mean $m_t^{(N)}$ and covariance $\Sigma_t^{(N)}$

Mean-field limit:

$$\mathrm{d}\bar{X}_t = \text{(linear dynamics)} + \bar{\mathsf{K}}_t(\,\mathrm{d}\bar{Z}_t - \frac{1}{2}H(X_t^i + \bar{m}_t)\,\mathrm{d}t), \quad \bar{X}_0 \sim p_0$$

$$\bar{\mathsf{K}}_t = \bar{\Sigma}_t H^\top$$

with mean-field mean $\bar{m}_t = \mathsf{E}[\bar{X}_t|\mathcal{Z}_t]$ and covariance $\bar{\Sigma}_t = \mathsf{Cov}(\bar{X}_t|\mathcal{Z}_t)$

Error analysis:

1. Analysis of the mean-field system
2. Analysis of the converegnce of the finite-N system to the mean-field limit

Finite-N system $\overset{(2)}{\approx}$ mean-field system $\overset{(1)}{=}$ Kalman filter

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Error analysis of the EnKF

Assumption The system is stable and the observation matrix is full rank.

Error analysis

Under the assumption, EnKF admits the following error estimates:

$$\mathsf{E}[|m_t - m_t^{(N)}|^2] \leq \frac{\text{(const.)}}{N}$$

$$\mathsf{E}[|\Sigma_t - \Sigma_t^{(N)}|^2] \leq \frac{\text{(const.)}}{N}$$

where the constant does not depend on time.

Question:

- Kalman filter is stable when the system is stabilizable and detectable
- Can we prove uniform error estimates of EnKF under these conditions?

P Del Moral, J Tugaut. On the stability and the uniform propagation of chaos properties of ensemble Kalman–Bucy filters, 2018

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Conclusion

Part I: FPF is generalization of Kalman filter

Kalman filter $\longrightarrow$ Ensemble Kalman filter $\longrightarrow$ Feedback Particle filter

Part II: Analysis of FPF

Design	**Approximation**	**Error analysis**
Mean-field limit $\bar{X}_t, \bar{U}_t$	$\longrightarrow$	Finite-N system $\{X_t^i, U_t^i\}_{i=1}^N$

- Design: Infinitesimal optimal transport maps
- Approximation: Galerkin and Diffusion map approximation
- Error analysis: Convergence in linear Gaussian setting under strong conditions

Question: The three aspects are disjoint. Can they be carried out in a single framework?

Thanks for your attention!

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