Data-Driven Approximation of Stationary Nonlinear Filters with Optimal Transport Maps

Presented at
IEEE Conference on Decision and Control, Milan, Italy, Dec. 2024

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Joint work with Bamdad Hosseini & Amirhossein Taghvaei

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Dec 16, 2024

Numerical example: Dynamical model

$$X_t = \begin{bmatrix} \alpha & \sqrt{1-\alpha^2} \\ -\sqrt{1-\alpha^2} & \alpha \end{bmatrix} X_{t-1} + \sigma V_t$$

$$Y_t = h(X_t) + \sigma W_t$$

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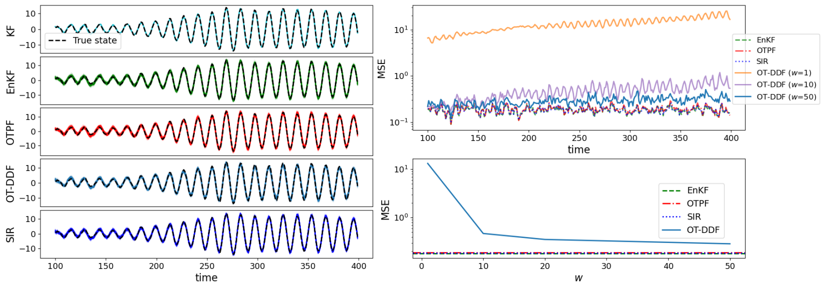
$$Y_t = h(X_t) + \sigma W_t$$

$$h(X_t) = X_t(1)$$



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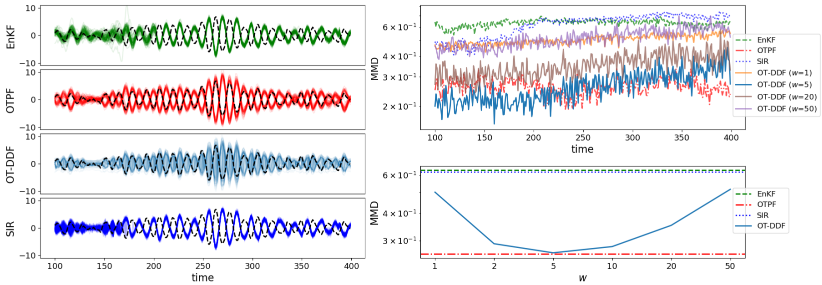
$$Y_t = h(X_t) + \sigma W_t$$

$$h(X_t) = X_t(1)^2$$



Numerical example: Lorenz 63 model

$$\dot{X} = f(X), \quad X_0 \sim \mathcal{N}(\mu_0, \sigma_0^2 I_3),$$
$$Y_t = x(1) + W_t, \quad W_t \sim \mathcal{N}(0, \sigma^2), \quad \Delta t = 0.01$$

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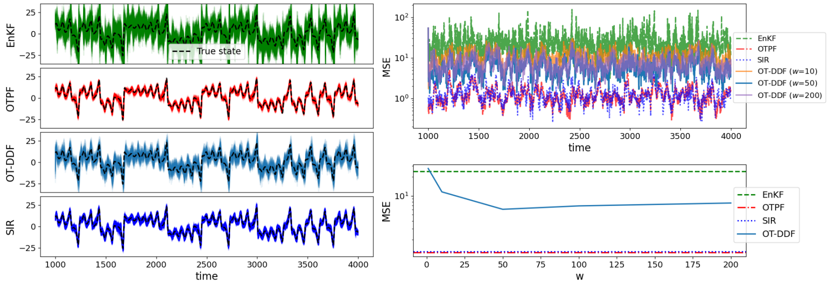

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Offline training time: 46.29 seconds

One-time step update:

Method	EnKF	SIR	OTPF	OT-DDF
time	1.7×10^{-4}	2.0×10^{-4}	6.8×10^{-2}	1.5×10^{-4}