Goal: introduce objectives of the course

Dynamical sys.:

- Study evolution of a system with respect to time.

Examples:

- Orbital motion of a Sattelite / rocket
- Training of a neural network
- Spread of infection in an epidemy
- Robot locomotion
- ... What are your examples?

- In order to describe a dyn. sys., we need two components:

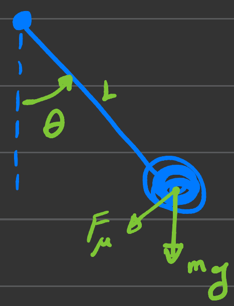
① State → min number of variables required to predict the future indep. of the past.

What is the state for the examples?
What is the state for stock market? chatGPT?

② update law → the mathematical rule that governs the update of the state

→ comes from physical principles and Data → sys. identification

Example: (pendulum)



- Newton's law:

$$mL^2\ddot{\theta} = -mgL\sin(\theta) - \mu L^2\dot{\theta}$$

$$\Rightarrow \quad \ddot{\theta} = -\frac{g}{L}\sin(\theta) - \frac{\mu}{m}\dot{\theta}$$ → 2nd order diff. eq.

- state: $X = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$ → min variables required to pred. the future.

- update law: we write the update law as a 1st order diff eq. of X

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$

$$\Rightarrow \quad \dot{X} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} X_2 \\ -\frac{g}{L}\sin(X_1) - \frac{\mu}{m}X_2 \end{bmatrix} =: f(X)$$

- we call $f(X) = \begin{bmatrix} X_2 \\ -\frac{g}{L}\sin(X_1) - \frac{\mu}{m}X_2 \end{bmatrix}$ the update law.

- General rep. of a dyn. sys (with a n-dim state)

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} f_1(x_1, \dots, x_n) \\ \vdots \\ f_n(x_1, \dots, x_n) \end{bmatrix} = f(x)$$

we usually drop the t dependence

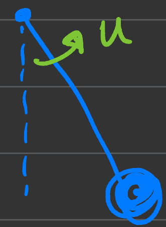
Controlled dyn. sys.:

control input (actuators)

$$\dot{x} = f(x,u)$$
$$y = h(x,u)$$

observation (sensors)

Example:



$$\dot{x} = \underbrace{\begin{bmatrix} x_2 \\ -\frac{g}{L}\sin(x_1) - \frac{\mu}{m}x_2 + \frac{u}{mL^2} \end{bmatrix}}_{f(x,u)}$$

$y = x_1$ ⇝ encoder the measures the angle

- What is this course about?

Analyze and Control long-term behavior of nonlinear dyn. systems.

questions
- what happens as $t \to \infty$?
- Is the sys. stable?
- how to design control law that leads to desired behavior?
- How does disturbances and uncertainties effect the result?

Illustration with pendelum

Course outline and Objectives:

① elementary methods for analysis of $\dot{x} = f(x)$

phase portrait → works for 1 or 2 dim sys.

linearization → only gives local behavior

② Fundamentals of diff. eq. (what does $\dot{x} = f(x)$ mean)

perturbation analysis $\dot{x} = f(x) + \varepsilon(t,x)$

③ Lyapunov method for stability

For nearly linear systems $\dot{x} = Ax + \varepsilon(t,x)$

Convergence regions and invariant sets

Gradient flow for optimization

④ Input-output stability (How does output change under disturbances?)

⑤ passivity (Stability of interconnected sys.)

⑥ Control Lyapunov functions and optimal cont.

design control law with stability guarantees.